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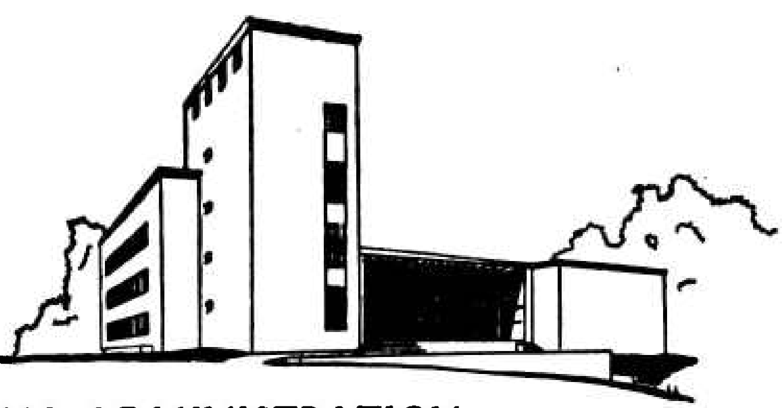

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Search-Theoretic Models of Organization Control by Budgeted Multiple Goals

by

A. Charnes and A. C. Stedry

This paper is a revised version of an earlier paper, "Some Models of Organization Response to Budgeted Multiple Goals," ONR 60, Northwestern University, The Technological Institute. The research for this paper was partially supported by the Management Sciences Research Group, Carnegie Institute of Technology, under Contract Nonr 760(24), NR 047-048 and by the project Temporal Planning and Management Decision Making under Risk and Uncertainty at Northwestern University, under Contract Nonr 1228(10), Project NR 047-021, both with the Office of Naval Research. Reproduction of this paper in whole or in part is permitted for any purpose of the United States Government. We are indebted to W. W. Cooper for comments and suggestions.

Abstract

Models of situations in which individuals are faced with multiple activities among which they must allocate their effort are postulated. Optimal allocations are found for four assumed motivational structures - profit maximization and three involving performance goals in each of the activities. Heuristic approximations to the optimal allocations are developed. Also, the relationships between the various motivation structures are explored.

Introduction

This paper forms one part in a series of research studies concerned with budgetary theory and practice. In these studies, interest has centered on organizational aspects of budgeting with a concomitant emphasis on control and related ideas of goal formation, effort, motivation, superior-subordinate relationships and other components of goal-oriented activity, Thus, by emphasis, at least, these studies differ from the customary approaches to budgeting research which deal primarily with planning, forecasting and coordination. 1/ The latter have, perhaps naturally, been directed to the economic magnitudes which are deemed pertinent for electing more or less rational choices between alternatives, insuring that all alternatives are considered and, at least in principle, that optimization criteria or other *desiderata* are satisfied when a choice is finally made. By and large (it seems fair to say) the latter types of studies have tended to bypass -- or at least they have failed to formalize and make explicit -- the organizational, psychological, and managerial dimensions which require consideration in any attempt to characterize the control aspects of budgeting for purposes of scientific validation and testing.

We shall not report here the results of such empirical tests as we have been able to make although we shall provide references to this work at suitable points in the text. The emphasis here will be rather on the formal models which we have utilized as guides for effecting these studies and tests.

1/ See Charnes and Cooper [3] pp. viii and 39-40 for further, but brief, discussion of control and planning. A more extensive discussion which is closely related to the budgeting literature (and also relevant parts of the psychological literature) is contained in Stedry [18].

A few remarks on the models which we have developed -- as well as the way we have developed them -- are perhaps in order at this point. First, we observe that there is very little in the budgeting-management literature which provides clear guides for the formation of such models and related methods of analysis. Second, we observe that a similar situation prevails in the basic scientific literature in areas such as economics, psychology, and organization theory where the guidance has had to be garnered form ideas and constucts which are either vague, partially formulated, or tangetial to the main direction of this research on the control aspects of budgeting. This is true, for instance, of such constructs as allocation of managerial effort, organizational goals, and even true of the psychological "aspiration level" constructs which we shall also employ, but only after adjusting and extending them to the multiple-task situations in a probabilistic context with which we shall be concerned.

Some brief remarks on the mathematics we shall utilize are also in order. One possible approach would proceed entirely by means of the theory of sets and related ideas from formal logic, say, as a way to reduce substantially the number of assumptions utilized in order thereby to secure the utmost generality. Even if successful, however, such an approach would not have yielded the kinds of sharp analytical guides which were desired for the concimitant empirical research. A large and difficult task of interpretation would have remained, even at the end of such an analysis, if only because the guides from management, psychology, and economics are vague at crucial points. I.e., the researcher would then be left with the task of specifying some function in order, say, to formulate experimental hypotheses for the investigation of such phenomena as effort and aspiration relative to probabilistic and multiple-task situations with no more information than is available to these authors and would then be left with the task of carrying the mathematics forward to utilize the specific properties of the functions chosen. Hence a different approach with correspondingly more specialized assumptions was attempted.

One part of this approach is based on the ideas of chance-constrained programming [1] which is here extended so that cases in which the joint probability of simultaneously satisfying several constraints (in the form of goals) can be considered explicitly. This is further elaborated to include cases in which a subordinate, say, in response to different reward structures may strive to maximize his reward as a function of goal attainment subject to constraints on all, or a subset, of the probabilities of attainment of specific goals taken individually. From there it is an easy step to include stipulations on other subsets which must then be satisfied when pursuing the indicated maximization, and so on.

Of course, a chance-constrained programming approach generally requires some specification of the underlying probability distributions. A wide range of possible choices is thus available. For instance, one might make the usual assumptions of normality, etc. in order to obtain access to the prepared theory and tables of classical statistical theory. [2] The resulting properties of symmetry do not otherwise have much appeal, however, since assymetry (or skewness) is much more likely to prevail in the kinds of situations which are pertinent for this research. Doubtless other properties such as

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1/ We do not here cover the entire gamut of decision rule possibilities as discussed in Charnes and Cooper [4] dealing with this topic.

2/ For a chance-constrained programming formulation in such terms see Charnes and Cooper [5].

multi-modality and discontinuity are also present and should be considered as the situations examined and the related theories and methodologies emerge from further research in the area of control budgeting. For the present, we have deemed it advisable to adhere to fairly simple distributions which at least have the indicated skewness properties. These distributions may be regarded as of the exponential type -- they are also conditional distributions -- although they will also be seen to be related to the kinds of distributions utilized in the search theory literature. 1/

Although the latter theory is also concerned with effort allocation, at least in a generic sense, we do not wish to push the point of actual relevance. Suffice it to say, therefore, that this approach has at least proved useful in supplying guidelines for the empirical research which we have also undertaken. Furthermore, although we have had to extend these search theory models to multiple-goal situations we have also found it possible to do so in a way that does not yield unduly cumbersome and complicated models and results. Finally, we have also been able to effect the developments in such a way which transforms such dubious variables as "effort" into other, more easily observed, performance variables.

To close this introductory section, we also note that we have elsewhere treated other aspects of these models. 2/ Here, however, we shall attempt to close certain issues which were left unattended in these other treatments. In particular, we shall establish certain conditions for the existence of the indicated solutions. Also we extend and sharpen these previous formulations in other respects as well. Hence, in these respects at least, we can now be sure that analytical theory which we are using to guide these studies is of a logically consistent and non-empty variety although, of course, this does not

1/ See de Guenin [11] for a discussion of the bearing of this theory on "effort allocation" for its possible extensions to other kinds of statistical distributions and its origin in the work of B. O. Koopman.

2/ See Charnes and Stedry [7], and [8].

settle the issue of empirical validity for either normative or descriptive applications.

2. Basic Assumptions:

We shall return shortly to the nature of our specializing assumptions in the mathematical and statistical domains. Here, however, we want to bring to the fore the kinds of psychological and organizational assumptions we are also making, and we shall try to do this in a way which also permits references to pertinent parts of the literature in these domains.

First, we shall delimit the environment in which control is exercised by focussing on a hypothetical supervisor of a sub-unit in a hypothethical organization. We further assume that it is possible to obtain measures of performance which may be imputed to this supervisor in each of the activities in a set which is assigned to him. More precisely we shall be concerned with relative performance measures which are, in turn, functions of the amount of effort which this supervisor decides to assign to his various tasks. As already indicated, we shall effect these developments in a way which factors out "effort" 1/ as an independent variable so that we can use "performance" as a surrogate. This will also enable us to avoid having to deal explicitly, and at length, with such issues as the significance that may attach to "decisions on effort allocation" and how this kind of phenomenon is to be observed and measured.

Second, we shall also confine ourselves to short-run situations and phenomena. One reason for this is that the field studies — and laboratory

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1/ As used here the term "effort" is to be understood as "search effort" in the sense described by Simon [16] and March and Simon [15]. That is, the supervisor is not necessarily viewed as expending "toil" directly on the task but rather as expending effort in a search for improved solutions, prodedures, so that better performance may ultimately emerge from development of the resources—human and material—which are at his disposal.

studies, too -- have generally been restricted to just such short-run cases. Thus we shall abstract from such problems as aging, gaining experience, moving up (or down) the managerial hierarchy and related phenomena such as organization revision, and wearing out of resources.

It is, of course, necessary to assume some kind of technological environment which relates performance to effort. Part of this is accomodated by the kinds of search-distribution functions which we have already discussed. Thus, in particular, we want the relationship between performance and effort to be of a probabilistic nature. In particular, we deem it desirable to have some sort of "diminishing returns" hypothesis so that the probability of increased performance begins, at some point, to increase only at a diminishing rate with further effort. We also want the probability of attaining any performance goal to be, in general, less than unity even when performance and effort are positively related.

The multiple-task situations which are of interest here will naturally lead to considerations of the possible mixes of performance that a supervisor's sub-unit may produce. We shall want to relate these (and the supervisor's responses) to various reward structures, and thereby also relate these to possible incentive schemes and maximizing propensities of a (more or less) classical economics variety. For instance, we shall be concerned with the managerial considerations that might attach to various reward mixes which might be used to produce performance mixes that correspond to what is desired by higher management.

These kinds of reward-performance possibilities are certainly pertinent to the problems of budgetary management. They are not sufficient of themselves however, to provide what is wanted for purposes of budgetary control unless one is willing to make very strong simplifying assumptions on the informational and behavioral variables which are likely to be present. For instance, the kinds of instructions that a subordinate receives on his physical outputs may enter in ways that color or qualify his behavior relative to the dimensions of the monetary rewards he may be interested in attaining. We shall, in any event, assume that this is the case. We shall also assume that the way a management has responded to his performances mixes in the past will also influence his current behavior. Finally we shall assume that this hypothetical supervisor is also capable of forming goals, more or less independently, and that these too will influence the performance of his sub-unit through the indicated kinds of technological relations and possibilities.

Although we shall assume that management is free to change its instructions and its rewards in ways that can affect the supervisor's performance via his relative effort allocations we shall also make certain other assumptions which also affect the total effort that this supervisor will expend on all tasks. Thus, for instance, we shall assume a stipulated maximum amount of effort at his disposal. Although this will be stated in the form of an inequality constraint, the maximizing assumptions that we also make will (as we shall see) have the effect of ensuring that all of this available effort is actually allocated.

Given these assumptions and their consequences, then, our strategy will be directed to deducing propositions which, interpreted as predictions, can then be used to test the model relative to situations in which the following phenomena are present: (1) a single and fixed technology and (2) several motivational structures of prescribed kind relative to (3) various kinds of short-run instructions and reward structures that a management might utilize to influence the performance which such a supervisor might induce in his sub-unit.

These predictions are then available for empirical test.[1] Within the confines of this theoretical paper, however, we shall go a step further by outlining an example of a method for proceeding to control system design should one of the motivation structures provide predictions which can be empirically validated. Specifically, we shall consider the following kinds of motivational assumptions: (1) maximization of expected reward where reward is proportional to expected profit; (2) maximization of expected reward where reward is a function of the attainment or non-attainment of a set of goals; (3) maximization as in (2) with constraints (lower bounds) placed on the probability of attainment of a set of minimum standards; and (4) maximization of the joint probability of attainment of goals in all performance areas. For each of these assumptions we shall determine optimal behavior including the effort allocation to each performance area and the expected performance. From these optimal determinations we shall derive approximate heuristics which could constitute the basis for behavioral predictions where limitations on computational ability are assumed to render the precise optima unattainable by an actual supervisor.

1/ The question of whether or not anything but behavior may be objectively measured has been a long-argued one in psychology. The problems involved in attempting to determine underlying motivation structures by direct measurement are such that the selection of motivation structure which provides good predictions of behavior as the basis for the design of systems to influence behavior may constitute the only operational way to proceed even if it does not supply adequate understanding of the underlying phenomena.

Technological Assumptions

In order to construct a framework within which the effects of the various motivational assumptions may be studied it is necessary to assume some function which relates performance and effort in each activity. For reasons that we have indicated above it is desirable that this function be one in which the probability of attainment of any performance level (above some arbitrarily low level) be less than unity for finite effort, and that the function portray diminishing returns to additional effort as effort is increases.

Specifically, we assume that performance in the j^{th} activity, x_j, can take on discrete values a_{ij}, $i=0,1,\ldots,\infty$, where $i > \ell \Longleftrightarrow a_{ij} > a_{\ell j}$. Further we assume that the probability of x_j attaining a value greater than or equal to a_{ij} may be expressed as a function of ρ_j, the amount of effort allocated to the j^{th} area, by the equations:

(1a) $$P(x_j \geq a_{oj}) = 1 \quad \text{for } j=1,\ldots,n$$

and

(1b) $$P(x_j \geq a_{ij}) = k_{ij}(1-e^{-\alpha_j \rho_j}) \quad \text{for } i=1,\ldots,\infty;\ j=1,\ldots,n$$

where

$$i < \ell \Rightarrow k_{ij} \geq k_{\ell j}.$$

$$0 \leq k_{ij} \leq 1$$

$$k_{\infty j} = 0$$

$$\alpha_j \geq 0$$

It is readily perceived that the attainment probability approaches k_{ij} asymptotically as ρ_j increases. The rate of approach to k_{ij} is determined by the parameter α_j; the greater α_j, the less the amount of effort required to obtain the same proposition of the limiting probability of attainment.[1] The resemblance to certain functions used elsewhere in the context of search theory is of course, clear. However, the specification of two parameters as we have done here provides for considerably greater flexibility than is provided by these earlier functions.[2] It is possible to differentiate between levels of performance difficult to attain because (1) k_{ij}, and hence the probability of attainment is small no matter how much effort is consumed; even if α_j is large and (2) α_j is small thus requiring much effort to come close to k_{ij} even if the latter is close to or equal to 1. The difference may be illustrated by an analogy to two games; in one the player receives a nickel if a toss of a fair coin (already in hand) comes up heads; in the other he receives the nickel if he finds it in an elaborate Chinese puzzle-box. The probability of earning at least five cents cannot exceed one-half in the first case although little, if any, effort is required to reach the limiting probability. In the second case, the likelihood of earning at least five cents is high, although a considerable amount of effort may be expended in the process.

For some of the motivation structures to be investigated it is only necessary to specify the probability of attainment of one, or at most a few,

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1/ In fact, its reciprocal in analogous to the "time constant" of electrical engineering. Specifically $1/\alpha_j$ is the amount of effort required for a probability of attainment approximately equal to 63% of the limiting probability.

2/ E.g., Charnes and Cooper [4], in our terminology, assume $k_{ij}=1$ and $\alpha_j = \alpha_k$ for all j, k.

performance levels as expressed above. Where the motivation is in terms of expected values, however, something must be known about the density of x_j over its entire range.[1/] We note that:

(2a) $$P(x_j = a_{0j}) = 1 - k_{1j}(1 - e^{-a_j \zeta_j}) \qquad \text{for } j = 1, \ldots, n$$

and

(2b) $$P(x_j = a_{ij}) = (k_{ij} - k_{i+1,j})(1 - e^{-a_j \zeta_j}) \qquad \text{for } i = 1, \ldots, \infty;\ j = 1, \ldots, n$$

The sum of the density terms over i is, of course, equal to unity and the expected value may be expressed as:

(3) $$E(x_j) = a_{0j}\left[1 - k_{1j}(1 - e^{-a_j \zeta_j})\right] + \sum_{i=1}^{\infty} a_{ij}(k_{ij} - k_{i+1,j})(1 - e^{-a_j \zeta_j})$$

Noting that this sum may be partitioned into constant term and a term in ζ_j, we let:

(4) $$\tilde{x}_j = \lim_{\zeta_j \to \infty} E(x_j) = a_{0j}(1 - k_{1j}) + \sum_{i=1}^{\infty} a_{ij}(k_{ij} - k_{i+1,j})$$

so that the expected value may now be simplified to:

(5) $$E(x_j) = \tilde{x}_j - (\tilde{x}_j - a_{0j})\, e^{-a_j \zeta_j}$$

Higher moments of this distribution, if desired, may be analogously derived.[2/]

1/ It is quite possible that the occurrence of a particular motivation structure may be influenced by the relative estimability of expected values or other distribution parameters which require some knowledge of the distribution over its entire range as opposed to probabilities of attainment at a few specific points.

2/ Specifically, if we define:
$$\tilde{x}_j^{(n)} = \lim_{\zeta_j \to \infty} E(x^n{}_j)$$
we find:
$$E(x_j^n) = \tilde{x}_j^{(n)} - (\tilde{x}_j^{(n)} - a_{0j}^n)\, e^{-a_j \zeta_j}$$

We shall investigate the implications of possible behavioral responses to organizational controls within the framework of this technology. Although the functional forms found for the allocative responses are thus limited, the insights gained are not. The methodology to be used is generally applicable to technologies in which the probabilities of goal attainment (1b) and the expected values (5) are concave, differentiable functions in ρ_j. Thus when (and [illegible]) a technological form having these properties is found which is more applicable to a particular situation, it may readily be substituted for the functional form we have selected.

Model I - Maximization of Expected Profit

The usual assumption made in research on the intrafirm control system in a decentralized firm is that each individual supervisor will (or should) act to maximize profit. The relationship between the profit maximization model and others is therefore needed if a transition is to be provided between economic theory and the control mechanisms actually observed in business practice. The primary feature which distinguishes this model from others we shall present is the absence of perceived discontinuities in reward associated with the attainment of specific levels of performance. The absence of such discontinuities allows for tradeoffs between performances in the various activities which are both determinant and finite -- a property which holds at only a few points in other models.

We assume here that the supervisor perceives his reward to be proportional to the profit contributed by his sub-unit, and that he wishes to maximize the expected value of his reward. Assume that unit of performance, x_j, contributes in amount β_j to profit,[1] independent of S_j. Then his reward may be expressed as:

$$R = \sum_{j=1}^{n} \beta_j x_j \tag{6}$$

and its expectation:

$$E(R) = \sum_{j=1}^{n} \beta_j E(x_j) \tag{7}$$

[1] The multiplication of all of the β_j by a constant - e.g., to convert profit into a proportional reward - leaves the problem unchanged. For convenience, we will assume profit and reward here to be equal although, as pointed out by Whinston [20] this is rarely the case in actual decentralized organizations.

For the technology assumed, we may express this expectation as a sum over the terms of (5), *viz.*,

$$E(R) = \sum_{j=1}^{n} \beta_j \tilde{x}_k - \sum_{j=1}^{n} \beta_j (\tilde{x}_j - a_{oj}) e^{-a_j \rho_j} \quad (6)$$

The first summation is independent of the effort allocation, and indeed, of total effort expended by the supervisor. We shall term it the *technological optimum*; it may be interpreted as the expected value of profit (and reward) *if the supervisor's efforts were unnecessary*. It represents the expected profit that could be attained by the organizational sub-unit were it possible to assume that the factors of production respond automatically, without supervisory intervention, to the requirements of profit maximation.[1] The second summation represents the detraction from the technological optimum which would be attained under the assumptions of classical economic theory where the factors of production are cooperative.[2] In order to maximize profit (and his expected reward) the supervisor wishes to allocate his effort so as to minimize this sum of detractions occasioned by his inability to expend infinite effort. In particular, assuming his total effort expenditure to be limited to an amount ρ, his reward maximization may be expressed as:

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[1] It is related to the concept of "perfect standard" in the budgeting literature - i.e., a performance that can be attained if no scheduling difficulties, errors, accidents, etc. occur - which is generally viewed as the technologically optimal performance were the "human element" not to intervene.

[2] Or, in the budgeting context, departure from "perfect standard".

Minimize:

$$\sum_{j=1}^{n} \beta_j \left(\bar{x}_j - a_{oj}\right) e^{-a_j \rho_j} \tag{9}$$

Subject to:

$$\sum_{j=1}^{n} \rho_j < \rho$$

We shall present the results here without proof, having developed the algorithm for the solution of the mathematical problem elsewhere [7]. 1/ Specifically the supervisor should order the products $a_j \eta_j$ largest to the smallest where:

$$\eta_j = \beta_j \left(\bar{x}_j - a_{oj}\right) \tag{10}$$

He should then allocate all of his effort to a subset of the activities, J, where $a_j \eta_j$ is sufficiently large. "Sufficiently large" is determined by the inequality:

$$\min_{r \in J} \hat{\gamma}_r > \frac{1}{\sum_{j \in J} \frac{1}{a_j}} \left[\sum_{j \in J} \frac{\hat{\gamma}_j}{a_j} - \rho \right] \geq \max_{s \notin J} \hat{\gamma}_s \tag{11}$$

where $J = \{ j \mid \rho_j > 0 \}$

$$\hat{\gamma}_j = \ln \left(a_j \eta_j\right) \tag{12}$$

The optimal effort allocations are given by

$$\rho_r = \frac{1}{a_r} \left(\frac{\rho}{\sum_{j \in J} \frac{1}{a_j}} \right) + \frac{1}{a_r} \left[\hat{\gamma}_r - \frac{\sum_{j \in J} \frac{\hat{\gamma}_j}{a_j}}{\sum_{j \in J} \frac{1}{a_j}} \right] \quad \text{for } r \in J \tag{13}$$

1/ The results follow from the Kuhn-Tucker theorem [[illegible]] and an extension of an algorithm of Charnes and Cooper [[illegible]].

and

(14) $$\ell_s = 0 \qquad s \notin J$$

In this form our earlier statement regarding the multiplication of all of the β_j by a constant leaving the effort allocation unchanged may be readily verified. Recalling (10) and (12),

(15) $$\hat{\gamma}_j = \ln [\alpha_j \beta_j (\tilde{x}_j - a_{oj})]$$

Then, if we multiply each of the β_j by a constant, e^k to obtain β_j' we observe:

(16) $$\hat{\gamma}_j' = \ln[\alpha_j \beta_j' (\tilde{x}_j - a_{oj})] = \ln[\alpha_j \beta_j (\tilde{x}_j - a_{oj})] + k = \hat{\gamma}_j + k$$

$$\frac{\sum\limits_{j\epsilon J} \frac{\hat{\gamma}_j'}{\alpha_j}}{\sum\limits_{j\epsilon J} \frac{1}{\alpha_j}} = \frac{\sum\limits_{j\epsilon J} \frac{\hat{\gamma}_j}{\alpha_j}}{\sum\limits_{j\epsilon J} \frac{1}{\alpha_j}} + \frac{k \sum\limits_{j\epsilon J} \frac{1}{\alpha_j}}{\sum\limits_{j\epsilon J} \frac{1}{\alpha_j}} = \frac{\sum\limits_{j\epsilon J} \frac{\hat{\gamma}_j}{\alpha_j}}{\sum\limits_{j\epsilon J} \frac{1}{\alpha_j}} + k$$

The multiplicative transformation clearly leaves the inequality (11) and the equations (13) unchanged.

We may therefore express the behavioral predictions in quite simple form. If we let

(17) $$\beta_j' = \beta_j e^k \qquad \text{where} \quad k = \frac{\ell}{\sum\limits_{j\epsilon J} \frac{1}{\alpha_j}}$$

we may express the optimal effort allocations as:

(18) $$\ell_r = \frac{1}{\alpha_r} \ln[\alpha_r \beta_r' (\tilde{x}_r - a_{or})] \qquad \text{for } r \epsilon J$$

and

$$\ell_s = 0 \qquad \text{for } s \notin J$$

Substitution of (18) in (5) we obtain:

(19)
$$E(x_r) = \hat{x}_r - \frac{1}{\alpha_r \beta_r'} \qquad \text{for } r \in J$$

$$E(x_s) = a_{0s} \qquad \text{for } s \notin J$$

Thus the performance expected at optimum for activities receiving effort will differ from the infinite effort expected value by an amount inversely proportional to the product of the sensitivity to effort and the relative contribution. For those not receiving effort, the expected performance will be at the level attainable with no effort.

We have thus found a simple heuristic for behavioral prediction provided the technological [illegible], [illegible], [illegible] be [illegible] along with the no-effort performances at the other extreme, a_{0j}.

Model II - Reward as a Function of Goal Attainment 1/

The concept of goal, as used here, is related to the psychological aspiration level -- i.e., the level of performance whose attainment is perceived as constituting "success" and non-attainment, "failure". 2/

The role of such discrete levels of performance in "decision-making" context has been explicitly formulated by Simon [16]. He has postulated that an individual engages in satisficing as opposed to maximizing behavior. Performance in a task may be "acceptable" or "unacceptable" -- in the latter case an individual will search for alternative ways of accomplishing the task until an acceptable performance is found. The decision to search or not is assumed to be go-no-go. The value 3/ of performance is assumed to be 1 at or above the acceptable level, 0 elsewhere. It is argued that satisficing reduces considerably the amount of information the individual requires about the world -- e.g., the expected contribution to performance of all the possible alternatives that could be elicited by search -- thus becoming a viable behavioral pattern under the assumption that human rationality is bounded.

1/ This model is a modification of one we have presented elsewhere [7]. In this treatment, however, we will attempt to simplify the mathematical development in order to focus on the relationship of the model to existing theory as well as present heuristic interpretations not developed in the earlier paper.

2/ This construct has been formalized in Lewin et.al. [14] which treats the psychological concept of valence (roughly attractiveness) in a manner closely resembling utility maximization as has been pointed out by Starbuck [17]. The psychological focus, however, has been on the choice of the aspiration level rather than on decision affecting behavior subsequent to the aspiration level formation.

3/ Which may be interpreted as utility, if desired.

The satisficing concept has been extended to a multi-task environment in an organization - theoretic context by March and Simon [15] and Cyert and March [10]. They postulate, as part of a behavioral theory of the firm, the formation by the ruling coalition of an organization (or one of its sub-units) of a set of goals -- performances designated as acceptable -- for various activities. The organization, unable to compute its long-run expected profit (or utility) under assumptions of bounded rationality, formulates these short run targets to provide operational bases for decision-making.

The behavioral theory extends to the assumption that if performance in one of the activities appears to be falling short of the target, search effort will be concentrated in that activity. The theory does not, however, provide for the allocation of effort between two such activities or even which will be attended to first if a sequential process is envisioned. More broadly, the decision as to how to allocate effort to all of the activities where goals are assumed to exist for future (as yet unrecorded) performance has not been considered.

To this end we wish to superimpose on (and perhaps do injustice to) the satisficing theory a function which combines the various goal attainments into a single reward figure. We assume that a relative weight may be associated with the attainment of each goal. We extend the bounds on rationality to the extent that the probability of attainment of each goal as a function of effort allocated to the activity is assumed to be known. It should be pointed out that this last requires the computation of only one point on the density over the entire range of possible performance.

We postulate a goal, g_j, in each of the activities and a relative importance, weight, w_j, associated with attainment of the $j^{\underline{th}}$ goal. We define a goal-attainment operator, z_j, such that:

$$(20) \qquad z_j = \begin{cases} 1 & \text{where } x_j \geq g_j \\ 0 & \text{where } x_j < g_j \end{cases}$$

establishing a dichotomy between acceptable and unacceptable performance. The organizational sub-unit supervisor perceives his reward to be determined by the total of his goal attainments weighted by their relative importance, or: 1/

$$(21) \qquad R = \sum_{j=1}^{n} w_j z_j$$

We number the activities so that for $j = 1, \ldots, m$, a $g_{i(j)j}$ for some $i(j) > 0$ is selected and denoted by g_j; similarly the associated $k_{i(j)j}$ is denoted by c_j. For $j = m+1, \ldots, n$ we select $g_j \leq g_{0j}$, goals which may be attained without effort. The probabilities of attainment are then given by:

$$(22) \qquad P(x_j \geq g_j) = \begin{cases} c_j(1 - e^{-a_j \ell_j}) & , \; j = 1, \ldots, m \\ 1 & , \; j = m+1, \ldots, n \end{cases}$$

The expected value of z_j is then:

$$E(z_j) = \begin{cases} (1)\,[P(x_j \geq g_j)] + (0)\,[P(x_j < g_j)] = c_j(1 - e^{-a_j \ell_j}), & j = 1, \ldots, m \\ 1 & j = m+1, \ldots, n \end{cases}$$

1/ In a manner analogous to that described for Model I, performance and effort allocation may be shown to be invariant under a multiplicative transformation of the w_j.

Recalling that we assume in this model that the sub-unit supervisor wishes to maximize his expected reward, we note that:

$$(23) \qquad E(R) = \sum_{j=1}^{n} w_j E(z_j) = \sum_{j=1}^{n} w_j c_j (1-e^{-a_j \rho_j}) + \sum_{j=m+1}^{n} w_j \ .$$

Since the supervisor will not gain from effort allocation to activities $j=m+1,\ldots,n$, these and other constant terms may be eliminated yielding the problem:

$$(24) \qquad \text{Minimize:} \quad \sum_{j=1}^{n} w_j c_j e_j^{-a_j \rho_j}$$

$$\text{Subject to:} \quad \sum_{j=1}^{n} \rho_j \leq \rho$$

$$\rho_j \geq 0, \; j = 1,\ldots,n$$

which is in a mathematical form identical to that of Model I.

We define:

$$(25a) \qquad \hat{f}_j = \ln (a_j w_j c_j)$$

and

$$(25b) \qquad J' = (j \,|\, \rho_j > 0)$$

Then we renumber the activities so that $\hat{f}_1$ is the largest, $\hat{f}_2$ the next, and so on. The selection of the favored set, J', is determined by the inequality:

$$(26) \qquad \min_{r \in J'} \hat{f}_r > \frac{1}{\sum_{j \in J'} \frac{1}{\alpha_j}} \left[\sum_{j \in J'} \frac{\hat{f}_j}{\alpha_j} - \ell \right] \geq \max_{s \notin J'} f_s$$

The algorithm for solution proceeds by testing $J' = \{1\}$, $J' = \{1,2\}$, etc. until a J' is found which satisfies the inequality. Having determined the set, the optimal effort allocations are given by:

$$(27) \qquad \begin{cases} \ell'^{*}_r = \frac{1}{\alpha_r} \left[\hat{f}_r - \frac{1}{\sum_{j \in J'} \frac{1}{\alpha_j}} \left(\sum_{j \in J'} \frac{\hat{f}_j}{\alpha_j} - \ell \right) \right] & \text{for } r \in J' \\ \ell'^{*}_s = 0 & \text{for } s \notin J' \end{cases}$$

Clearly the solution is too complex in its exact form to expect that an individual will consciously behave precisely in this fashion. Simplifications are possible with additional assumptions. We shall provide one as an example. Suppose the products $(\alpha_j w_j c_j)$ are distributed according to a special form of the Pareto distribution such that:

$$(28) \qquad (\alpha_j w_j c_j) = (\alpha_1 w_1 c_1)\, j^{-1}$$

Then, suppose that, in testing $J' = \{1\}$, $J' = \{1,2\}$, etc., rather than using the quantity between the inequalities in (26) he uses, respectively, the lower bounds, $\hat{f}_2$, $\hat{f}_3$, etc. Thus with this approximation, and denoting by $\hat{f}_t$ $\max_{s \notin J'} \hat{f}_s$ the ℓ^*_r in (27) becomes simply

(29) $\tilde{\rho}'^{*}_{r} = \frac{1}{w_r}[\hat{r}_r \quad \hat{r}_t]$

Note that $\rho'^{*}_{r} \leq \tilde{\rho}'^{*}_{r}$ so that with this rule the effort is expended on possibly fewer tasks than in the exact solution. Taking antilogarithms in (29) and substituting from (28) we obtain

(30) $e^{a_r \tilde{\rho}'^{*}_{r}} = \frac{r}{t}$

A simple heuristic is thus established. If the t^{th} activity is the one with the largest $(a_j w_j c_j)$ not to receive effort, then effort will be allocated such that the r^{th} activity receives effort to the extent necessary to attain the r^{th} goal with probability $(1 - \frac{r}{t})$ times the limiting probability. Thus, if all of the available effort is allocated to three activities then the goal in the first activity will have an attainment probability of $\frac{3}{4} g_1$, the second $\frac{2}{4} g_2$, and the third $\frac{1}{4} g_3$. Whether or not this kind of heuristic will be learned or adopted by an individual may be tested in a laboratory experiment where the relative sizes of the $(a_j w_j c_j)$ may be determined at will. In a field situation, the form or forms of heuristics specific to the actual or perceived values would need to be developed for test. The case given here is merely demonstrative of a possible way to proceed from an optimizing scheme based upon the existence of goals to a heuristic which produces an approximate solution to the optimization problem whose simplicity makes it credible as a candidate for actual behavior.

In the satisficing context, the interpretation of the solution merits additional consideration. It is assumed in the behavioral theory of the firm [1] and in business writing generally that supervisory effort tends to be sufficiently limited that it is the "exceptions" or unsatisfactory

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1/ as developed in Cyert and March [10].

performances that receive attention. In its simplest form, the solution suggests a partitioning of activities into two sets--those which receive effort and those which do not. 1/ For those activities which initially had goals above zero-effort performance and do not receive effort it would seem that satisfactory performance for them has been redefined to be zero-effort performance.

We might therefore propose a conjecture whose investigation is beyond the scope of this paper, viz that the goal-setting process is at least a two-stage process. The initial goals are weighed against the effort available for attaining them by an optimization or heuristic method. The subset of activities for which goal striving is represented by receipt of effort, are then taken to be the goals set.

1/ Clearly a dichotomy between recipients and non-recipients of effort could hold for a large class of assumed technological or reward functions in addition to the one used here.

Model 3 - Reward Maximization Subject to Constraints on Individual Actitities

It has been suggested that the psychological aspiration level is not single-valued but rather, that an individual has, with respect to a given task, two or more levels at which he perceives discontinuities in satisfaction 1/. In the industrial control environment frequently several standards are found to exist for the same task - e.g., a "perfect" or "ideal" standard, and "attainable" standard, a "basis" standard - each carrying a different connotation as to the desirability, or necessity, of attainment. In military intelligence, one level of information is categorized as "need to know", a higher level as "nice to have".

One could, of course, characterize these differences in terms of a goal g_j whose attainment is perceived as associated with reward w_j, a higher goal g_j' with reward w_j', etc. Obviously the number of goals and rewards can be increased until, at the limit, every level of performance that can be recognized (i.e., every a_{ij}) is associated with a particular reward. If all of the incremental rewards are proportional to the performance increments with which they are associated, the resulting reward maximization problem is identical to that of profit maximization.

1/ For example, Gardner [12], p. 65 suggests:

> Might not an individual in a task such as dart-throwing entertain at one and the same time a wild hope that he will make a perfect hit and a more prudent hope that he will at least hit the target, with perhaps an additional, self-conscious hope that he will not appear too awkward in the eyes of the experimenter? In other words, is there not considerable likelihood that an individual's [illegible]s on a given trial are manifold, fluctuant, ephemeral, and differing qualitatively as well as quantitatively, with those aims which involve a specific score often giving way to aims which cannot possibly be described in terms of score values?

In any event, extension of the model in this direction is trivial, at least with the technology assumed. [1/] Furthermore, extensions of this kind do not allow for qualitative differences in the way such multiple levels may be treated.

Take, as an example, a student who defines as "good performance" the grade of A in two of his courses and B in three others. Let us further suppose that one of the latter is a course which is relatively unimportant to him - say, far removed from his major field - and attainment of a B difficult as well. Under assumption of the straightforward reward-maximization problem he might well arrive at a solution which allocates no effort to this course. Presuming the zero-effort solution to be a failing grade, it seems reasonable to assume that although it might interfere with his good performances in other courses the student would not wish to fail this one. More generally, he might wish to make sure that his effort allocation assures him of a small probability of failure in each course, regardless of the importance to him of good performance in the course.

In the industrial environment, similar examples of qualitatively different treatments of activities are perceived. Success or failure of a

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[1/] It may be shown that, for a set of goals g_k, $k=1,\ldots,P$, associated with a set of rewards such that w_{kj} is the non-negative incremental reward obtained by attaining g_k rather than g_{k-1} where

$$P(x_j \geq g_{kj}) = c_{kj}\,(1-e^{-\alpha_j \rho_j}),$$

it is merely necessary to substitute the quantity

$$\sum_{k=1}^{P} w_{kj}\, c_{kj}$$

for $w_j c_j$ in the reward maximization problem shown in the previous section.

supervisor in meeting a production deadline rarely appears on a budget evaluation; if anything the costs incurred by reason of the failure are generally reflected in another supervisor's accounts. Yet, meeting production deadlines frequently seems to be taken as a "given" ; such items seem to have a dominant role which cannot be readily perceived with relative weighting schemes. 1/ They seem to have the characteristics of a sine qua non or pure constraint whose attainment is necessary before other measures become relevant.

There are, of course, several ways in which such qualitative differences might be represented. As an example we present the following (simple) representation in which the supervisor sets limits on the probability of performances falling below some "minimum acceptable" levels, d_j. These are assumed to exist in a subset of the activities in which he is striving to attain "good" performance levels, g_j. 2/

1/ In interviews conducted by one of the authors in connection with a field study [19] foremen queried about their goals indicated some variant of "well, the production schedule has to be met - and no if's about it" but could not put a relative weight on this target in comparison to other goals. Nevertheless, they seemed to associate the attainment of other goals - usually cost and quality - with the "score-cards" their superiors kept on their activities for retention and promotion decisions.

2/ If some of the activities are not perceived as having goals which contribute positively to his rewards beyond attaining the minimum acceptable level, they may simply enter the reward function with zero weight and some dummy goal - e.g., the minimum acceptable level.

We assume the reward function and terminology of Model II except that, for simplicity, we ignore the possibility of $g_j = a_{oj}$ or $d_j = a_{oj}$ and assume that

(31) $$P(x_j > d_j) = c_j^{\prime}(1 - e^{-a_j \rho_j})$$

where $c_j^{\prime}$ is the k_{ij} associated with the performance level a_{ij} corresponding to d_j. For convenience, we assume that the n activities are numbered initially so that the first m of them have minimum acceptable levels. Defining $(1-\xi_j)$ as the maximum risk of non-attainment of d_j the supervisor is willing to take, we may state the reward maximization problem as in (24) with adjoined constraints, viz.,

(32) $$\text{Minimize:} \quad \sum_{j=1}^{n} w_j c_j e^{-a_j \rho_j}$$

$$\text{Subject to:} \quad c_j^{\prime}(1-e^{-a_j \rho_j}) \geq \xi_j, \quad j=1, \ldots, m$$

$$\sum_{j=1}^{n} \rho_j < \rho$$

$$\rho_j \geq 0, \quad j=1, \ldots, n.$$

Although straightforward application of the Kuhn-Tucker conditions is possible, the following transformation simplifies the problem considerably. Let $\bar{\rho}_j$ be defined by the equations

$$(33) \qquad c_j\,(1 - e^{-a_j \bar{\rho}_j}) = \xi_j \qquad j = 1, \ldots, m$$

$$\bar{\rho}_j = 0 \qquad j = m+1, \ldots, n$$

Then we effect a transformation of the ρ_j such that:

$$(34) \qquad \rho_j' = \rho_j - \bar{\rho}_j, \qquad j = 1, \ldots, n$$

The adjoined constraints are satisfied if and only if non negativity holds for the ρ_j'. Defining

$$(35) \qquad \rho' = \rho - \sum_{j=1}^{n} \bar{\rho}_j$$

it is clear that for the problem to be feasible, $\rho' > 0$. The transformed problem may be stated as:

$$\text{Minimize:} \quad \sum_{j=1}^{n} w_j c_j e^{-a_j \rho_j'}$$

$$\text{Subject to:} \quad \sum_{j=1}^{n} \rho_j' \leq \rho'$$

$$\rho_j' \geq 0, \; j=1, \ldots, n.$$

The mathematical solution is, of course, identical to that of Model II.

The behavioristic interpretation of the effort allocation decision made is best viewed as a two-stage process. First, the supervisor allocates all of the effort necessary to satisfy the "musts" - the minimum performance requirements he perceives to exist. Then he allocates whatever is left over to reward maximization. 1/ Some sort of mechanism of this kind is necessary to explain the occurrence of situations where increased difficulty in one activity - e.g., an extremely tight production schedule - seems to cause a whole series of budget "exceptions" (failures) without a corresponding increase in reward associated with the activity. Under the unconstrained reward maximization model, a sufficiently great increase in difficulty would drive the activity from the set of activities receiving effort unless its contribution to reward were very high. If the weight were high, however, one would expect a considerable amount of effort devoted to the activity generally, even in periods when attainment is easy, with a resulting high expected performance. The last is not generally observed; things like production schedules seem to be met consistently but exceeded rarely.

1/ A similar development, and similar interpretation, can be constructed for the profit maximization model, i.e. suppose a supervisor in a decentralized firm is given profit maximization instructions but (as seems to occur in practice) is also expected to adhere to certain policies presumed necessary for coordination of the activities of the decentralized units. Under these conditions it might be expected that he would allocate whatever effort is left over from the satisfaction of specific policy objectives (whose attainment is easier to measure) at minimum risk to profit maximization (whose measurement is more difficult). If the policy objectives are difficult to satisfy, the effort allocated to profit maximisation may be small and profit low. Such a phenomenon could explain the existence of profit objectives in addition to (or instead of) profit maximization instructions. Should other policies be incompatible with sufficiently high profits - an infeasible problem - than a re-evaluation of other policies might result rather than a continuation of a policy structure which simply produces low profits.

Model IV - Maximization of Probability of Attainment of All Goals

In this model, we go still one step further in the direction of reducing the importance of relative weights attached to goal attainment. Specifically, we assume "good performance" to be defined as attainment of all goals. This is particularly relevant to the situation where the goals are viewed as minimum specifications without whose attainment the entire product is considered unacceptable.

Take, for example, the specifications for an airplane. Among other things, minimum top speed, minimum range, minimum payload, etc., are specified in advance. No increase in top speed above this maximum, however, could compensate, say, for an inability to take off with a pilot aboard (assuming this to be a manned aircraft).1/

In the control environment context to which we have related our other models, a situation may be comprehended in which a superior's policy may be stated as "no exceptions will be tolerated." Such a policy was followed until quite recently in the promotion of military officers; only officers whose efficiency reports were perfect throughout their careers could be recommended for early promotion. The business executive who asks for a report of explanation from every supervisor who failed to make any budget is following a similar, if somewhat less severe, policy of

1/ Another plane, designed to be unmanned might be designed to fly faster but the specifications for this one call for a pilot.

penalizing one or many deviations equally.

Under any of these conditions only attainment of all goals will suffice. Hence, maximizing the probability of acceptable performance - the probability of attainment of all goals - maximizes expected reward. Assuming, for simplicity, the various goal attainments to be independent, the joint probability of attainment may be expressed as:

$$\Omega = \prod_{j=1}^{n} P\,(x_j \geq g_j) = \prod_{j=1}^{n} c_j(1-e^{-a_j \rho_j}) = (\prod_{j=1}^{n} c_j)[\prod_{j=1}^{n} (1-e^{-a_j \rho_j})] \tag{37}$$

Maximizing Ω is clearly equivalent to minimizing the quantity:

$$\mathcal{C} = -\ln\left(\frac{\Omega}{\prod_{j=1}^{n} c_j}\right) \tag{38}$$

so that the problem may be stated as

$$\begin{aligned} &\text{Minimize:} \quad -\sum_{j=1}^{n} \ln(1 - e^{-a_j \rho_j}) \\ &\text{Subject to:} \quad \sum_{j=1}^{n} \rho_j \leq \rho \end{aligned} \tag{39}$$

An explicit solution for the ρ_j is not available readily, if at all. An approximation to the explicit solution is available which makes use of the fact that unless the probabilities of non-attainment of the individual goals are quite small the product of the attainment probabilities would be meaningless as a criterion function. I.e., what is the meaning of maximizing the probability of acceptable performance where, say, it is in the neighborhood of 10%? The derivation of the approximate solution is given in the appendix.

Specifically, the optimal effort allocation is given by:

$$\rho_j = \frac{1}{\alpha_j} \ln \left(\frac{\mu + \alpha_j}{\mu} \right) \qquad , j = 1, \ldots, n$$

(40)

$$\tilde{\mu} \leq \mu \leq \tilde{\mu} e^{(1/2 - \sqrt{1/4 - \beta \tilde{\mu}})} \text{ 1/}$$

where

(41) $$\tilde{\mu} = \exp \left[- \frac{1}{\sum_{j=1}^{n} \frac{1}{\alpha_j}} \left(\rho - \sum_{j=1}^{n} \frac{1}{\alpha_j} \ln \alpha_j \right) \right]$$

and

$$\beta = \frac{\sum_{j=1}^{n} \frac{1}{\alpha_j^2}}{\sum_{j=1}^{n} \frac{1}{\alpha_j}}$$

Because the failure to attain any goal would result in a zero possibility of acceptable performance, all activities must receive effort. The requirement that the non-attainment probability in each activity be small is essentially a requirement that μ be small relative to each of the α_j or, alternatively, that ρ be large relative to $\sum_{j=1}^{n} \frac{1}{\alpha_j} \ln \alpha_j$ as may be seen in (41).

1/ This upper bound for μ is approximately $\tilde{\mu}(1 + \beta\tilde{\mu})$. Thus $\tilde{\mu}$ as an approximation to μ is in error by no more than $\beta\tilde{\mu}^2$.

Where μ is quite small, $\tilde{\mu}$ is a good approximation to μ and $\frac{a_j}{M}$ to $\frac{\mu+\alpha_j}{\mu}$. Examining this case, we note:

$$\rho_j \approx \frac{1}{\alpha_j} \ln \alpha_j + \left[\frac{1}{\sum_{j=1}^{n} \frac{1}{\alpha_j}} \left(\rho - \sum_{j=1}^{n} \frac{1}{\alpha_j} \ln w_j \right) \right], \; j=1, \ldots, n \tag{42}$$

This approximation is identical to the solution of the reward maximization problem of Model II where ρ is large and the η_j are all equal to 1. In the current problem, equal η_j are tantamount to equal weights, w_j, since, for the same reasons that require μ to be small relative to α_j, the c_j must all be close to 1 for the criterion to be meaningful.

Thus we would expect similar behavior to result from a "no exceptions" policy and a policy of not distinguishing between exceptions where: (1) the amount of effort available is sufficiently large that all activities would receive effort; and (2) the limiting probabilities in all activities are close to unity. The major difference between the two policies would seem to be that "counting exceptions" would remain a viable policy if total effort should be substantially reduced or some of the goals increased substantially in difficulty in situations where the "no exceptions" policy would cease to be viable. There is a safety valve in one -- the ability to drop an activity--that does not exist in the other. Assuming that an individual could be motivated to respond to either policy, the choice between them would seem to be dictated by whether it was desirable to drop a subset of activities or to drop all goal attainment probabilities generally if difficulty in one or more goals increases relative to the effort available.

Relationships Among the Models

At the beginning of this paper we noted the possibility of a difference between the desires of higher management and the criterion function to which a supervisor might actually respond. If such a disparity exists, it is in the interests of higher management to translate, if possible, its criterion function in such a way that the supervisor satisfies it (perhaps approximately) by satisfying his own.

It is generally assumed, in economic treatments of control in decentralized systems,[1] that the supervisors of the decentralized units will (or should) maximize the profits of their individual units. The resulting overall performance will then be optimal provided certain conditions on the interrelationships between the units obtain.[2] Suppose, however, that the unit supervisors can respond to goals, but do not maximize profit. The control problem may then be stated in terms of designing a set of goals which will produce the same effort allocations--and performances--as profit maximizing behavior.

Recalling the mathematical forms of the functionals to be minimized in the expected profit maximization and reward maximization models, viz.,

$$\sum_{j=1}^{n} \beta_j (\tilde{x}_j - a_{oj}) e^{-\alpha_j \rho_j}$$

(43) and

$$\sum_{j=1}^{n} w_j c_j e^{-\alpha_j \rho_j}$$

1/ See, for example, Arrow [1].

2/ A thorough treatment will be found in Whinston [20].

an intuitive formulation is clear. If the rewards, w_j, and goals, g_j (and hence the associated c_j), are chosen such that, for the activities receiving effort, (in the profit maximization solution),

$$w_r c_r = k\,[\beta_r(\tilde{x}_r - a_{ir})],\ r \in J \tag{44}$$

and, for those not receiving effort in the profit maximization solution,

$$w_s c_s \leq k \max_{j \notin J} [\beta_j(\tilde{x}_j - a_{oj})],\ s \notin J. \tag{45}$$

That is, to assure that effort is not allocated to an activity the rewards and/or limiting probabilities of attainment for need only be sufficiently small. A simple expedient which minimizes the amount of communication required would be setting all of the $w_s = 0$.

A more rigorous statement of the necessary and sufficient conditions is proved as a theorem in Appendix 2. These formal relationships are less interesting, however, than the relative simplicity of the translation. Similar translations can be developed for the other models. We have already alluded to the similarity between the chance-constrained reward maximization problem of Model III and a similarly constrained profit maximization model. Also, the equivalence between the approximate solution to the Model IV problem and a reward maximization problem with equal weights attached to each activity expands the scope of possible translation.[1/]

1/ Note, however, that translation of profit maximization for a supervisor responding to a "no exceptions" policy is severely limited unless some way of partitioning the activities may be found such that $\beta_j(\tilde{x}_j - a_{oj}) = \beta_k(\tilde{x}_k - a_{ok})$ for all j, k. The frequent occurrence of "no exception" policies--e.g., in government purchase contracting--may attest to difficulties involved in specifying criterion weights or performance contributions. Alternatively, a reexamination of the usefulness of such contracting procedures may be suggested.

Some writers have suggested the desirability of educating managers to profit maximizing behavior. An obvious alternative suggested by the relationships investigated here is the translation of the desired behavior into the motivational framework of the supervisor--if such can be established--rather than attempting to modify that framework.

Conclusions

We have investigated optimal behavior in a variety of assumed motivational situations and the relationships between them. We have developed heuristic approximations to some of the optimization procedures which can serve as bases for behavioral predictions in actual situations. An interpretive limitation should be noted, however. We have assumed throughout that an individual's behavior will conform to that which maximizes some criterion either exactly or approximately. Empirical validation of the behavioral predictions would not validate maximization as a motivational drive or assumption but rather the predictive usefulness of the model for design of control systems. The latter would be analogous to the usage of the principles of least action, least constraint, etc. in the physical sciences without the ascription of a behavioral teleology to inanimate objects.

We have attempted to show how the maximizing models are related to certain "rules" of behavior which may be stated programmatically - e.g., "first work on the activity which has the largest product of return to effort and reward."

But greater specificity - e.g., when to start working on the second-- requires increasingly greater incorporation of sophisticated computational routines. We have thus, at best, taken only a step in the direction of predicting types of behavior which appear to be highly programmed but devoid of complex forms of computation. E.g , the department store buyer of Cyert and March [10] and Clarkson's [9] trust officer exhibit behavior which is highly predictable from the behavioral programs which they seemed to have developed for themselves. The relationship between these programs and an underlying motivational framework is not made explicit papers, however. Implicit in the design of control systems is the problem of altering behavioral programs, a problem whose solution could undoubtedly be facilitated if it were possible to alter behavior by setting appropriate goals rather than by altering the programs in detail.

Thus, an avenue suggested for future theoretical research is the search for models of motivation structure whose heuristic interpretations can be made more explicit--in terms of behavioral programs--but whose computational requirements are less demanding than those exhibited here. Obviously individuals do make a choice of the activity to be allocated effort first and when to turn attention to another; control systems utilizing goals in various activities continue to be observed. The further investigation of the effects of multiple-goal control systems on effort allocation programs with an emphasis on developing the heuristic solutions seems merited. 1/

At this point however, the models presented here have been and can be useful for providing behavioral predictions for empirical test. Empirical observations are necessary as a guide to the choice of direction for the search for motivational frameworks which provide promise of serving as a vehicle for controlling the development of behavioral programs.

1/ Two theoretical works which are suggestive of particular directions of attack are the non-Archimedean utility structures of Charnes and Cooper in [3] and the vector utilities derived in Charnes, Clower and Kortanek [2].

Appendix 1

We wish to find an approximate solution to the problem:

$$\text{Minimize} \quad -\sum_j \ln\left(1 - e^{-\alpha_j \rho_j}\right)$$

$$\text{Subject to:} \quad \sum_j \rho_j \leq \rho$$

where, throughout, the summation is understood to be over $j=1, \ldots, n$.
The Kuhn-Tucker conditions are 1/

$$\frac{\alpha_j e^{-\alpha_j \rho_j}}{1 - e^{-\alpha_j \rho_j}} = \mu \quad , \quad j=1, \ldots, n$$

$$\sum_j \rho_j = \rho$$

$$\mu \geq 0 .$$

Solving for ρ_j in terms of u, we obtain

$$\rho_j = \frac{1}{\alpha_j} \ln\left(\frac{\mu+\alpha_j}{\mu}\right)$$

$$\sum_j \rho_j = \sum_j \frac{1}{\alpha_j} \ln\left(\frac{\mu+\alpha_j}{\mu}\right) = \rho .$$

It will be recalled that the original statement of the functional was in terms of an overall probability of success. It seems reasonable that, in most situations, unless this probability is fairly large, the individual will reject the criterion and adopt some other; it seems unlikely that an individual will consistently engage in an activity where he perceives

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1/ Note that for $\mu \leq 0$, no solution obtains for finite ρ_j so that the condition on $\Sigma \rho_j$ must be satisfied as an equality.

his chances of success at much less than 1/2. For the problem to be meaningful, then, the components of the joint probability must be fairly close to 1, indicating that each of the $e^{-\alpha_j \rho_j}$ will be small, especially so when the number of activities is large. Thus the ratio $\frac{\mu+\alpha_j}{\mu}$ must be large or $\mu \ll \alpha_j$. It follows that:

$$\frac{\mu+\alpha_j}{\mu} \approx \frac{\alpha_j}{\mu},$$

suggesting an approximation. We note that

$$\rho = \sum_j \frac{1}{\alpha_j} \ln \left(\frac{\mu+\alpha_j}{\mu} \right) \geq \sum_j \frac{1}{\alpha_j} \ln \left(\frac{\alpha_j}{\mu} \right).$$

Let an approximation to μ, $\tilde{\mu}$, be defined by

$$\rho \equiv \sum_j \frac{1}{\alpha_j} \ln \left(\frac{\alpha_j}{\tilde{\mu}} \right).$$

Since μ is determined by $\rho = \sum_j \frac{1}{\alpha_j} \ln \left(1+ \frac{\alpha_j}{\mu}\right)$, smaller values of $\frac{\alpha_j}{\mu}$ are involved than in $\rho = \sum \frac{1}{\alpha_j} \ln \left(\frac{\alpha_j}{\mu} \right)$, hence $\tilde{\mu}$ must be a lower bound for μ. An explicit solution is easily obtainable for $\tilde{\mu}$, for

$$\rho = \sum_j \frac{1}{\alpha_j} \ln \alpha_j - \ln(\tilde{\mu}) \sum_j \frac{1}{\alpha_j}$$

$$\tilde{\mu} = \exp \left[- \frac{1}{\sum_j \frac{1}{\alpha_j}} \left(\rho - \sum_j \frac{1}{\alpha_j} \ln \alpha_j \right) \right].$$

We note further that:

$$\rho = \sum_j \frac{1}{\alpha_j} \ln \left(\frac{\mu+\alpha_j}{\mu} \right) \leq \sum_j \frac{1}{\alpha_j} \ln \left(\frac{\alpha_j}{\mu} e^{\frac{\mu}{\alpha_j}} \right)$$

since $\mu + \alpha_j \leq \alpha_j \, e^{\frac{\mu}{\alpha_j}}$.

We next form the preceding inequality on ρ as

$$-(\rho - \sum_j \frac{1}{\alpha_j} \ln \alpha_j) \geq (\ln \mu)(\sum_j \frac{1}{\alpha_j}) - \mu (\sum_j \frac{1}{\alpha_j^2}).$$

Dividing through by $\sum_j \frac{1}{\alpha_j}$, the left side of this new inequality becomes the expression for $\ln \tilde{\mu}$. Thus

$$\ln \tilde{\mu} \geq \ln \mu - \mu \frac{\sum_j \frac{1}{\alpha_j^2}}{\sum_j \frac{1}{\alpha_j}}$$

Defining

$$\beta \equiv \frac{\sum_j \frac{1}{\alpha_j^2}}{\sum_j \frac{1}{\alpha_j}}, \text{ this can be written } \tilde{\mu} \geq \mu e^{-\beta\mu}.$$

We note next that

$$\mu \geq \mu e^{-\beta\mu} \geq \mu(1 - \beta\mu).$$

Let $\bar{\mu}$ and $\bar{\bar{\mu}}$ be defined by $\tilde{\mu} \equiv \bar{\bar{\mu}}(1 - \beta\bar{\bar{\mu}}) \equiv \bar{\mu} e^{-\beta\bar{\mu}}$.

Since $e^{-\beta\bar{\bar{\mu}}} \geq 1 - \beta\bar{\bar{\mu}}$,

$$\bar{\bar{\mu}} \geq \bar{\mu}$$

so that:

$$\bar{\mu} e^{-\beta\bar{\mu}} \geq \tilde{\mu} = \bar{\bar{\mu}} e^{-\beta\bar{\bar{\mu}}} \geq \mu e^{-\beta\mu}$$

The function $y = x e^{-\beta x}$ is a strictly increasing function of x for $0 \leq x \leq 1/\beta$.

Thus if $1/\beta \geq \mu$, $\bar{\mu} \geq 0$, then from $\bar{\mu} e^{-\beta\bar{\mu}} \geq \mu e^{-\beta\mu}$,

it follows that $\bar{\mu} \geq \mu$.

Hence

$$\bar{\bar{\mu}} \geq \bar{\mu} \geq \mu .$$

Solving

$$\bar{\bar{\mu}}(1 - \beta\bar{\bar{\mu}}) = \tilde{\mu}$$

we obtain 1/

$$\bar{\bar{\mu}} = \frac{1 - \sqrt{1 - 4\beta\tilde{\mu}}}{2\beta}$$

which yields the implicit constraint $\tilde{\mu} \leq 1/4\beta$.

Solving

$$\bar{\mu}\, e^{-\beta\bar{\bar{\mu}}} = \tilde{\mu} ,$$

we obtain

$$\tilde{\mu} \leq \mu \leq \bar{\mu} = \tilde{\mu}\, e^{\left(\frac{1}{2} - \sqrt{\frac{1}{4} - \beta\tilde{\mu}} \right)}$$

The error of the estimate, μ , is thus seen to be bounded via

$$1 \leq \frac{\mu}{\bar{\mu}} \leq e^{\left(\frac{1}{2} - \sqrt{\frac{1}{4} - \beta\tilde{\mu}} \right)}$$

Recalling the functional in the original problem:

$$\Omega \equiv \prod_j P\left\{ s_j \geq g_j \right\} = \prod_j c_j \; (1 - e^{-a_j \rho_j})$$

Let

$$v \equiv \frac{\Omega}{\prod_j c_j} = \prod_j (1 - e^{-a_j \rho_j}) ,$$

wherein we suppose optimal values for the ρ_j. Thus,

1/ Since either root satisfies $\bar{\bar{\mu}} \geq \bar{\mu}$, the smaller as the closer approximation to μ is preferred.

$$1 - e^{-\alpha_j \rho_j} \leq 1 - \frac{\mu}{\mu + \alpha_j} = \frac{\alpha_j}{\mu + \alpha_j} ,$$

$$w = \prod_j \left(\frac{\alpha_j}{\mu + \alpha_j}\right) = \prod_j \left(\frac{1}{1 + \frac{\mu}{\alpha_j}}\right) \geq \prod_j \left(1 - \frac{\mu}{\alpha_j}\right) \geq 1 - \mu \sum_j \frac{1}{\alpha_j} ,$$

$$\frac{1}{w} = \prod_j \left(1 + \frac{\mu}{\alpha_j}\right) \geq 1 + \mu \sum_j \frac{1}{\alpha_j} ,$$

$$\frac{1}{1 + \mu \sum_j \frac{1}{\alpha_j}} \geq w ,$$

$$\frac{1}{1 + \tilde{\mu} \sum_j \frac{1}{\alpha_j}} \geq \frac{1}{1 + \mu \sum_j \frac{1}{\alpha_j}} \geq w \geq 1 - \mu \sum_j \frac{1}{\alpha_j} \geq 1 - \bar{\mu} \sum_j \frac{1}{\alpha_j} .$$

We have thus established lower and upper bounds on the estimate of μ (and hence estimated ρ_j) and on w, which provide limits on the estimated probability of acceptable performance.

Appendix 2

The conditions for equivalence of the effort allocations under profit maximizaation, ρ^*_r, and reward maximization allocations, ρ'^*_r, in response to goals, $g_j = a_{ij}$, will be formally stated. The terminology developed in the paper will be introduced without further (formal) definition.[1/]

<u>Theorem:</u> Necessary and sufficient conditions for $\rho'^*_r = \rho^*_r$, $r \in J$, $r \notin J'$ are choice of g_j and w_j such that

(i) $$g_r > a_{or} \quad \text{for all } r \in J$$

(ii) $$\hat{f}_r = \hat{\gamma}_r + \hat{g}, \quad r \in J$$

where $\hat{g}$ is an arbitrary constant,

(iii) $$\hat{f}_{s \notin J} \leq \frac{1}{(\sum_{j \in J} \frac{1}{\alpha_j})} \left[\sum_{j \in J} \frac{\hat{\gamma}_j}{\alpha_j} - \rho \right] + \hat{g}$$

and

(iv) $$\min_{j \in J} \hat{f}_r > \frac{1}{(\sum_{j \in J} \frac{1}{\alpha_j})} \left[\sum_{j \in J} \frac{\hat{f}_j}{\alpha_j} - \rho \right]$$

<u>Proof</u>

Condition (i) is clearly sufficient for J to be included in the set of the first m j's in the cut effected by (22). For necessity, suppose that for some $k \in J$, $g_k = a_{ok}$. Then $\rho'^{*'}_k = 0$; but by definition of J, $\rho^*_k > 0$ so that $\rho^*_k \neq \rho'^{*'}_k$, a contradiction.

1/ It will be recalled that $\hat{\gamma}_j = \ln \alpha_j \beta_j (\bar{x}_j - a_{oj})$, defined for the profit maximization problem and $\hat{f}_j = \ln(\alpha_j w_j e_j)$, defined for the reward maximization problem.

Condition (ii) implies

$$\frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left[\sum_{j\in J}\frac{\hat{\gamma}_j}{\alpha_j}-\rho\right]+\hat{g}=\frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left[\sum_{j\in J}\frac{f_j}{\alpha_j}-\rho\right]$$

so that conditions (iii) and (iv) together imply $J' = J$. Also

$$\rho_r^{*'} = \frac{1}{\alpha_r}\left[\hat{f}_r - \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left(\sum_{j\in J}\frac{\hat{f}_j}{\alpha_j}-\rho\right)\right]$$

$$= \frac{1}{\alpha_r}\left[\hat{\gamma}_r+\hat{g} - \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left(\sum_{j\in J}\frac{\hat{\gamma}_j}{\alpha_j}+\hat{g}\sum_{j\in J}\frac{1}{\alpha_j}-\rho\right)\right]$$

$$= \frac{1}{\alpha_r}\left[\hat{\gamma}_r - \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left(\sum_{j\in J}\frac{\hat{\gamma}_j}{\alpha_j}-\rho\right)\right]$$

$$= \rho_r^*$$

Conversely, $\rho_r^{*'} = \rho_r^*$ implies $J' = J$, for otherwise either for some $k\in J$, $\rho_k^* > 0$ and $\rho_k^{*'} = 0$ or for some $k \in J'$, $\rho_k^{*'} > 0$ and $\rho_k^* = 0$. Together, $\rho_r^{*'} = \rho_r^*$ and $J' = J$ imply

$$\left[\hat{f}_r - \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left(\sum_{j\in J}\frac{\hat{f}_j}{\alpha_j}-\rho\right)\right]=\left[\hat{\gamma}_r - \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\left(\sum_{j\in J}\frac{\hat{\gamma}_j}{\alpha_j}-\rho\right)\right]$$

$$\hat{f}_r - \hat{\gamma}_r = \frac{1}{\left(\sum_{j\in J}\frac{1}{\alpha_j}\right)}\sum_{j\in J}\left(\frac{\hat{f}_j-\hat{\gamma}_j}{\alpha_j}\right).$$

Hence $\hat{f}_r$ and $\hat{\gamma}_r$ differ by the same constant, say $\hat{g}$, for all $r \in J$. For the necessity of condition (iii) we introduce without proof the following lemma: 1/

1/ Proved in [7].

Lemma

If $\hat{f}_{m+1} > \dfrac{1}{\left(\sum\limits_{j=1}^{m} \dfrac{1}{d_j}\right)} \left[\sum\limits_{j=1}^{m} \dfrac{\hat{f}_j}{d_j} - \rho\right]$,

then $\hat{f}_{m+1} > \dfrac{1}{\left(\sum\limits_{j=1}^{m+1} \dfrac{1}{d_j}\right)} \left[\sum\limits_{j=1}^{m+1} \dfrac{\hat{f}_j}{d_j} - \rho\right]$.

Now then, suppose $\exists \hat{f}_t$, $t \notin J$ such that

$$\hat{f}_t > \frac{1}{\left(\sum\limits_{j\in J} \frac{1}{d_j}\right)} \left[\sum_{j\in J} \frac{\hat{\gamma}_j}{d_j} - \rho\right] + \hat{g} .$$

By condition (ii) and the lemma

$$\hat{f}_t > \frac{1}{\left(\sum\limits_{j\in J} \frac{1}{d_j} + \frac{1}{d_t}\right)} \left[\sum_{j\in J} \frac{\hat{f}_j}{d_j} + \frac{\hat{f}_t}{d_t} - \rho\right]$$

implying $t \in J'$, or $J' \neq J$, a contradiction.

Finally, suppose, for some $q \in J$,

$$\hat{f}_q \leq \frac{1}{\left(\sum\limits_{j\in J} \frac{1}{d_j}\right)} \left[\sum_{j\in J} \frac{\hat{f}_j}{d_j} - \rho\right]$$

By condition (ii)

$$\hat{\gamma}_q \leq \frac{1}{\left(\sum\limits_{j\in J} \frac{1}{d_j}\right)} \left[\sum_{j\in J} \frac{\hat{\gamma}_j}{d_j} - \rho\right]$$

implying $q \notin J$, a contradiction. Q.E.D.

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3 REPORT TITLE

Search-Theoretic Models of Organization Control by Budgeted Multiple Goals

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report, March, 1965

5 AUTHOR(S) (Last name, first name, initial)

Charnes, A., and Stedry, A. C.

6 REPORT DATE	7a TOTAL NO. OF PAGES	7b NO. OF REFS
March, 1965	48	20

8a CONTRACT OR GRANT NO. NONR 760(24) b. PROJECT NO. NR 047-048 c d.	9a ORIGINATOR'S REPORT NUMBER(S) Management Sciences Research Report No. 32 9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report) None

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13 ABSTRACT

Models of situations in which individuals are faced with multiple activities among which they must allocate their effort are postulated. Optimal allocations are found for four assumed motivational structures - profit maximization and three involving performance goals in each of the activities. Heuristic approximations to the optimal allocations are developed. Also, the relationships between the various motivation structures are explored.

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Technical Report, March 1965

5. AUTHOR(S) (Last name, first name, initial)

Charnes, A., and Stedry, A. C.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March, 1965	35	42

8a. CONTRACT OR GRANT NO. Nonr 760(24) b. PROJECT NO. Nr 047-048 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Management Sciences Research Report No. 31* 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) Systems Research Memorandum 125 (See item 11)

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13. ABSTRACT

Two approaches to organization theory having origins in economic theory are investigated for their relevance to problems of organization control. A third approach is explored via the construction of quasi-rational models of goal oriented behavior, incorporating features of both of the others. The need for such models is discussed as well as their general relevance to the pursuit of knowledge of the description and design of organization control systems.

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See items [illegible] and 11.

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